

## **Estimation and sizing of tank capacity by technical evaluation and optimization of storage-pump power plants operation in the power system**

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### **ABSTRACT**

*Storage-pump power plants flatten the load curve, and flat load curve actually improves technical parameters. Given the rising fuel curve, the increasing production cost at high powers (peak) is huge. So the difference of energy cost between peak and non-peak hours has to be equal to an amount that could cover the storage-pump power plant internal losses. In order to determine the optimal economic activity level of a storage-pump power plant, first, it is necessary to consider its economic evaluation. The issue of economic evaluation of a storage-pump power plant actually goes back to electricity generation compromise in the non-peak load region and the ultimate cost of electricity generation in the peak load region. Therefore, the current value of total daily income during operation of this technology has to be maximized. It has to be noted that the receiving from network (input energy), delivering to the network (generated energy), and the ultimate cost of electricity generation are a function of time and vary in different days of year. Production and consumption planning has to be in a way that minimizes the cost of thermal power plants. After determining the optimal time level and the amount of production and consumption of storage-pump power plants in a charge and discharge cycle, a method using the proposed technique has to be suggested in order to determine optimal tank capacity. This method is actually a suggestion about the economic optimum sizing of a storage-pump power plant, taking into account the costs of similar power plants and transmission losses.*

*Keywords: storage-pump power plant, peak and non-peak hours, tank capacity, technical optimization, economic optimization.*

### 1. Introduction

Increasing energy consumption all around the world as a result of increased population, technology development, etc. makes the need to develop the sources of energy production, especially at peak hours, more critical. Quick response to adjust the network frequency and voltage, especially in the time of incidents, as well as making a balance between energy production and consumption, and so network frequency and voltage control, are the determining factors in network stability [1]. Thermal power plants exhibit slow response regarding the changes in the required energy, so they make significant losses in the efficiency. In contrast, hydroelectric power plants quickly come into operation without compromising the efficiency and provide the possibility to meet requirements [2, 3]. In this regard, and in order to store energy in large scale and with high quality, there are direct and indirect methods to store the surplus energy in non-peak hours to produce more energy at peak hours [4, 5]. In recent decades, a set of technologies have been developed and used to store energy in the power network and [6] generally discusses the applied and feasible storage methods and elements. Increasing the initial cost of establishing new power plants and increasing the public concerns regarding environmental issues, as well as rising energy prices, storage units have attracted increasing attention. Storage-pump power plants, compressed air energy storage, flywheels, and batteries are some of the examples, to name a few [4, 5]. The optimal physical positioning of storage elements is explored in [7]. Classification of different types of storage elements and their physical nature is discussed in [8], where the storage-pump power plants are mechanically considered in the energy storage section in long-term. [9] deals with economic arguments regarding different types of storage elements, as well as the objective function of maximizing their benefits. The results of [9] reveals that storage via compressed air energy storage (CAES) method is the closest method of storing electricity to the storage-pump power plants. In [10], the electrical energy storage system (EES) applied to a commercial building is analyzed. Besides dealing with the market demands and economic evaluations, it evaluates the social advantages, namely power quality, reliability, and operation. Storage-pump power plants store the electrical energy as potential energy of water. This system is composed of two tanks. At peak hours, when the cost of energy is high, energy is generated and in low load times, when the energy demand decreases, the inexpensive energy is used to pump water from the lower tank to the upper one [11, 12]. Meanwhile, the issue which is of great importance is energy storage. In order to plan for a storage-pump power plant, one has to decide about the timing and amount of water to be pumped to the upper tank, as well as timing and amount of water to be released to generate energy [13, 14].

**Table 1. Details of the parameters used in the research**

$MC_g[n]$ : the cost of electricity generation in $n^{th}$ hour in generation mode	$IC_{tot}$ : income and earnings of storage-pump power plant in a daily cycle
$MC_p[n]$ : the cost of electricity generation in $n^{th}$ hour in pumping mode	$G_t[n]$ : total production power of thermal power plants
$t_n \in [i-l]$ : number of hours the power plant works in generation mode	$P[n]$ : total power consumption of network
$t_n \in [j-k]$ : number of hours the power plant works in pumping mode	$E$ : consumed energy in period T
$PH_g[n]$ : power of storage pump in generation mode	$P_{max}$ : the maximum load in a period
$PH_p[n]$ : power of storage pump in engine mode	$T$ : the desired period, that could be a day, week, month, or year
$n$ : represents different hours of a day (e.g. $n=2$ represents (1-2) o'clock)	$m$ : the number of power plants
$S[n]$ : sum of the amount of overflow water, the evaporated water, and capacity of water leakage in the dam	$X[n]$ : capacity of the upper tank in $n^{th}$ hour
$Q[n]$ : sum of the capacity of input water to the upper tank (e.g. a stream), and the amount of input water to the upper dam due to raining on it	$IC_{incom}$ : the income gained by using storage-pump power plant
$D[n]$ : outlet flow rate of the dam	$P_e$ : the power of installed power plant
$a_x, b_x, c_x$ : the coefficients of cost function of thermal power plants	$P_{av}$ : the average consumed load
$PG_{x,max}$ : the maximum output power of the thermal unit $i$	$P_a$ : nominal power of the thermal unit
$PG_{x,min}$ : the minimum output power of the thermal unit $i$	$P_g$ : the overall power that network consumers have
$a_1, a_2$ : constant values [28, 29]	$p_l$ : total system loss
$\eta$ : the whole cycle power plant efficiency	$PG_i$ : output power of $i^{th}$ unit
$G_{crit}$ : critical power point of the whole system	$h(P_{Gi})$ : indicating the inequality constraint on produce power limitation
$MC_{crit}[n]$ : thermal cost equivalent to $G_{crit}$	<b>storage</b> : the minimum rate power changes of storage-pump power plant

Since water has to be pumped to the upper tank after producing energy in this power plants, the storage-pump power plants produce energy in a limited period of time. In addition, their operating cost is much higher than the traditional hydroelectric power plants due to pumping cost [15].

Storage-pump power plant planning as a way to increase production in renewable power plants is discussed in [16], aiming to consider an optimal plan by the whole system. Given the random nature of

renewable resources and the load, the problem of storage-pump power plant capacity depends on the planning for these power plants.

In [17], a new coordination procedure of wind power plants and storage-pump power plant is proposed based on predicting the daily output of wind energy, while in [18], this approach involves developing an optimal control strategy for mounting a pair of wind power plant and storage-pump power plant, which is done using a stochastic model which estimates the maximum expected revenue in a planning horizon.

In [19], a two-stage production planning model is presented taking into account the uncertainty of the power of wind power plant and storage-pump power plant. Comparing the two different stages of the planning model, the plan for the future days of storage-pump power plant is considered. The first stage includes saving the updated plan to flatten the supply/demand curve by the thermal unit, while the second stage is responsible for reducing the error of wind power estimation based in the real wind power scenarios.

In [20], hydroelectric storage-pump power plants are used instead of batteries in independent solar and wind hybrid systems. A comparison between thermal and storage-pump power plants in [21] reveals that storage-pump power plants are capable of improvement in all production performance characteristics of thermal power plants. Presence of stochastic parameters, namely energy cost in electricity market, in [22, 23] resulted in widespread use of optimization methods to estimate the required capacity or determine the optimal scenario for operation of a storage-pump power plant. [24] optimized the desired objective function using Genetic Algorithm and proposed a new approach to manage the modeling of hydropower plant operation optimization. Planning in a multi-purpose framework consisting of two objective functions was modeled in [25]. Although the role of storage-pump power plants was considered in this paper, there is no analysis in the context of economic and technical issues of these power plants. [26] carried out storage-pump power plant modeling. For optimization, the profit objective function has to be maximized. Derivative method of setting the power changes/time changes ratio equal to zero gives the minimum sale price to gain the maximum profit. In fact, this paper offers a pricing method for storage-pump power plants. The object fiction in [27] is maximizing the profit of storage-pump power plant, and uses direct method with a proposed algorithm to determine its best pricing time. In [28], the UCP<sup>1</sup> is solved considering the planning constraints of a dynamic planning algorithm after entry and exit of a storage-pump power plant, and its profit in a power system is obtained from the difference of solving UCP in presence and absence of a storage-pump power. Although the profit can be calculated in this paper, the optimal plan is not performed for a storage-pump power plant, and it was considered as an infinite storage element. So if the capacity of a storage-pump power plant is assumed to be finite, there won't be the possibility of planning for the time and amount of its production and consumption. Besides solving UCP in presence of storage-pump power plants, [29] addressed the problem of thermal power plants pollution. An optimization method for operative planning of a storage-pump power plant up to a one-year horizon is presented in [30], in which the objective function is maximizing the profit and the optimization algorithm used is dynamic planning algorithm. Maximizing the objective function gives storage-pump power plant optimal planning. The storage-pump power plant planning in this paper is also not optimal. A new method for planning the operation pattern of a storage-pump power plant is presented in [31], considering the problem of surplus electricity, reducing the fuel cost, and reliability of power source. Given the reliability of power source, the evaluation model is estimated using Monte Carlo simulation, and the objective function minimizes the thermal production cost.

[32] discusses the various methods of optimizing hydropower plants and conducts a comparison between different optimization methods. However, none of the optimization methods used for storage-pump power plants are consistent with the reality, and are just a pure theory based on mathematical formulas. Furthermore, none of the papers mentioned above have calculated the profit of presence or absence of power plants in the optimal states, and the possibility of obtaining the real power is not mentioned in the. For example, none of these papers have mentioned the minimum power for production, and it hasn't been considered as an input for simulation. Power production of the generator also has a tap, which varies in different generators. Therefore, using just one mathematical method is not practical and feasible. [33] proposed a model to determine the optimal strategy considering the uncertainties in a hydroelectric power

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<sup>1</sup> *Unit Commitment Problem*

plant. The objective function in this paper was defined as the economic profit of a storage-pump power plant in a charge and discharge cycle, which has to be maximized based on the primary constraints of the problem and reality. Various formulaic methods were addressed in [32], however, no algorithm was defined in this paper based on the prevailing constraints. Technical and economic considerations were not considered simultaneously in the researches and studies mentioned above, and these two concepts have never been addressed along with each other, i.e., one research only considers the technical aspect, while another one just takes into account the economic aspect. But these two aspects complement each other, and studying one of them without considering the other one is just pure theory. This study presents a new algorithm for technical and economic optimal planning of a storage-pump power plant, which involves the best hours for a storage-pump power plant to enter and exit the desired network, as well as the amount of power production per hour and economic distribution among the thermal units based on these time intervals, so that the profit in a storage-pump power plant would be maximized. Another point that hasn't been considered yet is that when considering a power system with all of its thermal power plants, cost functions, and the losses in transmission lines between units, whose 24-hour load is known by different load prediction methods, what considerations have to be made, and how much storage capacity or what type of storage-pump power plant can be suggested, considering the economic and technical aspects, so that, first, the power plant operator would achieve the highest profit; second, the economic distribution of thermal units be in a way that minimizes the total cost of thermal power plants; and third, all technical constraints of the network and storage-pump power plant would be provided. For this purpose, and in order to improve the performance of a storage-pump power plant, an objective function and an algorithm is needed that could determine the best time of power plant entry into operation at generation mode, and the amount of power generation at peak time and high reservoir capacity at operation mode.

From economic perspective, the value of produced energy at peak time has to be way more than the amount of consumed energy in time of low load to compensate the efficiency (about 82%) of power plant.

The economic income of a storage-pump power plant can be analyzed from two perspectives:

1. Long-term profit
2. Short-term profit

In long-term economic profit, the income of a storage-pump power plant during several years is considered, and the effect of primary investment parameters and other costs, such as wear of the components and miscellaneous expenses are taken into account. Short-term income refers to a full engine and generator cycle that can be considered in a multi-day horizon, and is called the current operation profit from the storage-pump power plant.

## **2. Modeling of the problem parameters**

Various goals could be considered in operating a storage-pump power plant, each of which are defined by an independent objective function in the optimization stage. The following technical and economic objectives are concerned in this study:

1. Flattening the network load curve in the optimal state
2. Optimal planning to obtain the maximum profit from a storage-pump power plant and minimize the costs of thermal power plants

The first objective is valuable in technical and systemic terms, and the technical parameter which is improved by presence of a storage-pump power plant is discussed in the following. The second objective is specific to economic institutions and is in line with economic benefits of the power plant.

### **2-1. Technical considerations of storage-pump power plants**

The most important technical parameter improved in presence of a storage-pump power plant is the load factor (Equation (1)). It is one of the important characteristics of determining the consumption pattern and, therefore, the status of utilization of resources power plants. As the load factor approaches the unit value, the difference between the average load in Equation (5) and the maximum load decreases, which means making better use of the existing facilities, and the load curve also tends to be more uniform.

$$LF = (E / (P_{max} * T)) j = (P_{av} / P_{max}) j \quad (1)$$

Parameter "r" represents the reserve coefficient (Equation (2)). It has an inverse relationship with peak load, and decreasing peak by storage-pump power plant results in increasing the reserve coefficient.

$$r = P_e / P_{max} \quad (2)$$

Since different part of power plants need servicing and maintenance after a certain time of operation, as there are time when it is not possible to get load from them due to unexpected events, the capacity of the installed power plant network has to be higher than the maximum consumption load in order to maintain the continuity of supplying energy for the customers without any interruption. Parameter "n" in Equation (3) represents gain coefficient.

$$n = P_{av} / P_a \quad (3)$$

In Equation (4), parameter "g" is the contemporaneous factor. Presence of a storage-pump power plant reduces the contemporaneous factor.

$$g = P_{max} / P_g \quad (4)$$

The total power that the consumers possess in a potential network is much more than the maximum consumption power in a network as the consumers use electricity at different times, depending on their characteristics. As the contemporaneous factor decreases, energy supply takes place in a more economical way since contribution of energy suppliers requires less investment to establish new power plants and lines.

The parameter  $P_{av}$  in Equation (5) represents the average consumption load. Higher efficiency in storage-pump power plants increases the average load. If the efficiency is 100%, the average load will be constant.

$$P_{av} = E / t \quad (5)$$

Achieving the above objectives is not possible with one scenario, and the scenario associated with one objective is generally different from the other. Based on the network priorities at any time, the operator can choose the suitable objective function and determine the operation scenario.

## 2-2. Economic considerations of storage-pump power plants

Operating a storage-pump power plant has economic efficiency when the price of electricity at peak and non-peak times has a significant difference. In fact, the economic profit of a storage-pump power plant results from the difference of electricity price at peak and non-peak times.

Neglecting the miscellaneous expenses, for example the operative cost against the total costs saved in a storage-pump power plant, the daily income of a storage-pump power plant is calculated using Equation (6).

$$IC_{tot} = \sum_{n=i}^{n=l} (PH_g [n] \times MC_g [n]) - \sum_{n=j}^{n=k} (PH_p [n] \times MC_p [n]) \quad (6)$$

In fact, utilizing a storage-pump power plant is economic when not only the electricity price in time of peak load is higher than that at non-peak times, but also this difference has to compensate the  $(1 - \eta)$  percent losses.

## 2-3. The minimum profitability constraint of storage-pump power plants

According to Equation (6), the minimum sale price at peak load has to be consistent with Equation (7) in order not to have economic loss [31].

$$MC_{gen} [n] \geq \frac{\sum_{n=j}^{n=k} MC_p [n]}{(k - j)\eta} \quad (7)$$

Therefore, the sale price in Equation (7) is the price at operation point of a storage-power plant in order not to have economic loss.

So a formula or curve is needed to calculate the equivalent price and final costs of thermal power plants, given a certain power delivered, and vice versa. The results of calculations will be a rising curve with positive slope for any system or any kind of thermal, hydro, and atomic power plants.

**2-4. The rising curve of thermal power plant fuel cost**

At low powers, the final cost of electricity generation is lower due to presence of highly efficient power plants. So in order to meet system requirements, lower efficiency power plants have to gradually enter the power system. This would increase electricity price at higher demanded powers, and the price increases exponentially after a certain power value [34, 35].  $G_{crit}$  is the critical point power of the whole system, so that if the total network consumption exceeds  $G_{crit}$  (the critical point power of the power system in power plants other than storage-pump ones) and reaches the exponential part of the rising curve of fuel cost, the production of storage-pump power plants will economically efficient. Figure (1) shows the proposed algorithm to calculate the rising curve of fuel cost in a power system.

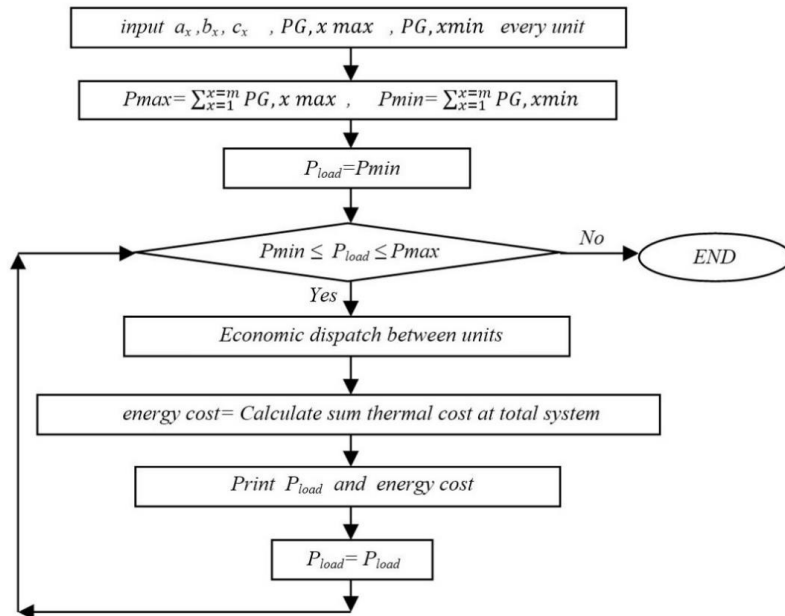


Figure 1. Algorithm of calculating the rising curve of fuel cost in power system

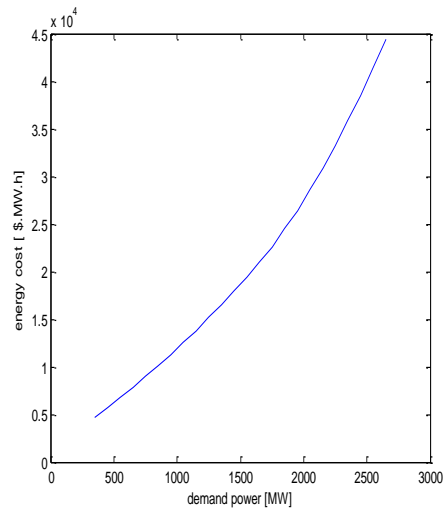


Figure 2. Rising curve of fuel cost

### 2-5. Equations of tank capacity and electrical power in storage-pump power plants

All of the pumping cycle-batch production, and input/output water flow rate to/from the system are assumed to be zero. Under this circumstance, Equation (8) is valid, where "D" is water flow rate, and "X" is the effective capacity of the upper tank. Positive sign for "D" means producing electrical energy [42, 43].

$$X[n+1] = X[n] - D[n] \cdot h - S[n] + Q[n] \quad (8)$$

$\Delta t = h$  is the time step, which is set to 1 hour.

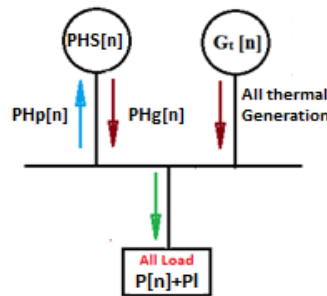
Equations (9) and (10) are used to calculate the power at production and pumping modes [36, 37]. Given these equations, there is a linear relationship between the power of the storage-pump power plant and water flow rate.

$$PH_g[n] = a_1 \cdot D[n] \quad (9)$$

$$PH_p[n] = \frac{\eta \cdot D[n]}{a_2} \quad (10)$$

The constraint on the instant balance and equality of electrical energy demand and supply in a power system from the perspective of storage-pump power plant modeling is given by Equation (11). Figure (3) is the electrical schematic of connection of a storage-pump power plant to the power system.

$$P[n] + P_l + PH_p[n] = PH_g[n] + \sum_{n=1}^{n=x} G_t[n] \quad (11)$$



**Figure 3. Electrical schematic of connection of a storage-pump power plant to the power system**

Storage-pump power plant cuts the peak load down, fills the low load pit, and flattens the  $G_t[n]$  curve. Amount of losses varies with changing the power of any unit.

### 2-6. The problem of technical and economic optimization of a storage-pump power plant

The objective function to optimize the storage-pump power plant planning is given by Equation (12) in case there is one storage-pump power plant with a known capacity. Solving the objective function below gives the optimal time and the amount of production and consumption.

$$\text{Max } IC_{tot} = \sum_{n=i}^{n=l} (PH_g[n] \times MC_g[n]) - \sum_{n=j}^{n=k} (PH_p[n] \times MC_p[n]) \quad (12)$$

The above objective function, in fact, aims to provide the optimal planning state considering all constraints. At higher parts of the load curve, it has to be in production mode by automatically maximizing the profit in a certain time period, given the amount of storage, while it has to be in pumping mode at the lower parts since the cost of energy is higher in peak areas of the load and lower in pit areas.

The constraints of the problem of technical and economic optimization of a storage-pump power plant are:

1.  $PH_p \min \leq PH_p[n] \leq PH_p \max$
2.  $PH_g \max \geq PH_g[n] \geq PH_g \min$
3.  $X \min[n] < X[n] < X \max[n]$

4.  $MC_g [n] > \frac{MC_p [n]}{\eta}$
5.  $MC_{critical} [n] = \frac{MC_p [n]}{\eta} \xrightarrow{\text{incremental cost diagram}} \mathbf{G}_{crit}$
6.  $p [n] > \mathbf{G}_{crit} - \mathbf{PH}_{gmax}$  at generation mode

If the constraint  $p[n] < G_{crit} - PH_{pmax}$  is not violated at pumping mode, it means that something has happened when the network load was low, so it's better to stop pumping. For example, consider a special day when an important soccer game is being displayed on TV in low load time of the power system. So the power system consumption is much more than the predicted amount, and a large load will be removed from the power system by stopping the pumping. The constraint  $p[n] > G_{crit} - PH_{gmax}$  in generation mode is also set in order not to have economical loss.

### 2-7. Economical distribution of thermal power plants in presence of a storage-pump power plant

The problem of economical load distribution can also be the optimization of a certain objective function, such as Equation (13), which, of course, has the performance limitations of units and the power system. In the following objective function, the power arrangement of thermal power plants in presence of a storage-pump power plant is determined by considering the losses among units.

$$\text{Min } F_T (P) = \sum_{i=1}^N F (P_{Gi}) \quad (13)$$

subject to

$$h_{min} \leq h(P_{Gi}) \leq h_{max}$$

Another challenging issue in modeling the real behavior of generators' cost arises when the unit has a number of different fuels. In this case, the cost function is modeled by a few pieces of quadratic functions, given in Equation (14), each corresponding to a specific thermal unit fuel.

$$f(P_i) = \begin{cases} a_{i1} + b_{i1}P_i + c_{i1}P_i^2 & \text{fuel1, } P_{i1}^{min} \leq P_i \leq P_{i1}^{max} \\ a_{i2} + b_{i2}P_i + c_{i2}P_i^2 & \text{fuel2, } P_{i2}^{min} \leq P_i \leq P_{i2}^{max} \\ \vdots \\ a_{ij} + b_{ij}P_i + c_{ij}P_i^2 & \text{fuelj, } P_{ij}^{min} \leq P_i \leq P_{ij}^{max} \end{cases} \quad (14)$$

Network loss is an exclusive function of output power of the units. This method, known as the loss relation method, is expressed by a quadratic function in terms of output powers of the generators. The simplest quadratic function is given by Equation (15).

$$P_l = \sum_{i=1}^{ng} \sum_{j=1}^{ng} P_i B_{ij} P_j \quad (15)$$

Equation (13) shows the operation cost function, which has to be minimized as the objective function for the problem of economical load distribution. The most important operation cost function is the cost function of thermal units' active power production, which is obtained by estimating the curve of cost of the consumed fuel in a thermal unit.

### 3. Algorithm of solving the optimization problem

The optimization algorithm is *PSO* Algorithm. Any solution for the problem in *PSO* is a bird in the search space, which is called *particle*. Any particle in n-dimensional space is considered as a low-capacity and low-weight mass. Each particle has a fitness value, which is determined by the fitness function of the problem, i.e., the bird closer to the food has a higher fitness.

#### 3-1. PSO algorithm implementation process

The problem performs the search by updating the location of the particles. Each particle in multi-dimensional space (depending on the problem requirements) is determined by two values of  $X_{id}$  and  $V_{id}$ ,



representing the spatial location and velocity of the  $d^{\text{th}}$  dimension of  $i^{\text{th}}$  particle, respectively. The dimension of space of the problem is equal to the number of parameters in the desired function to be used for optimization. The *PSO* particles are initially created randomly and iteratively move in the  $n$ -dimensional space of the problem to search the new possible options by calculating the optimality as an assessment criterion. The location of each particle is updated by two optimal value at each stage of movement of the population. The first value is the best answer in terms of fitness, known as *pbest*<sup>2</sup>, which has been obtained for each particle separately, while the other one is the best value obtained by all particles in the whole population so far, known as *gbest*<sup>3</sup>. These values appear in the natural process of group movement and represent the concepts of individuals' experience memory and the knowledge of positions of a leader or queen (the best particle), respectively. After finding the two values of *pbest* and *gbest* in any iteration of the algorithm, the velocity and location of each particle will be updated by the following equations.

Mathematically, a particle's position in an  $n$ -dimensional vector is expressed by Equation (16).

$$X_m = (X_{m,1}, X_{m,2}, \dots, X_{m,n}) \quad (16)$$

The velocity of this particle in an  $n$ -dimensional vector is also expressed by Equation (17).

$$V_m = (V_{m,1}, V_{m,2}, \dots, V_{m,n}) \quad (17)$$

It can be argued that the *PSO* basic algorithm solution is generally in the following order:

1. A set of random particles (solutions) are created. Then the position of each particle will be determined and its fitness will be calculated.
2. If the fitness obtained for the particle is the best value along the process, its location will be saved as *pbest*.
3. In any iteration, the particle with the best fitness value is selected among all particles, and its location will be saved as *gbest*.
4. The velocity of each particle is determined given the *pbest* and *gbest* positions.
5. The new position of the particles is determined.
6. If the ending conditions, such as the maximum iteration or the minimum error, are satisfied, the operation will end, otherwise, we go back to stage 2.

### 3-2. Hybrid modeling of thermal and storage-pump power plants

According to the fact that a storage-pump power plant has two modes of generation and engine, a separate algorithm is defined for each mode, however, the optimization algorithms are rather the same and complementary in the two kinds of power plants.

A parameter called *storestage* is introduced in the proposed method. It is practically the minimum production of a storage-pump power plant generator, which can increase or decrease in taps. The slope rate of a storage-pump power plant in both generation and engine modes is expressed by Equation (18).

$$\begin{cases} PHp[n] - PHp[n-1] \leq \text{storestage pumping} \\ PHg[n] - PHg[n-1] \leq \text{storestage generation} \end{cases} \quad (18)$$

In this study, it is assumed that:

$$\text{storestage generation} = \text{storestage pumping} = \text{storestage}$$

The *storestage* parameter is, in fact, representative of power quality type. As it gets smaller, the distortion of the load curve decreases. The value of this parameter depends on the type of synchronous machine in both engine and generation modes of a storage-pump power plant.

Introducing the *storestage* auxiliary parameter, the operation algorithm for a one-day period is as follows:

1. Entering the minimum value of a production mode generator [*storestage*]
2. Entering the load curve prediction at any certain time (the smaller time interval, the better) [ $P[n]$ ]

<sup>2</sup> Personal best

<sup>3</sup> Global best

3. Entering the efficiency, the maximum production of storage-pump power plant, and the maximum storable energy in the upper tank [ $X_{max}$ ,  $rand$ ,  $Phg_{max}$ ,  $PHp_{max}$ ]
4. Entering the cost parameters and the approximate production of thermal power plants [ $a_{ij}$ ,  $b_{ij}$ ,  $c_{ij}$ ,  $P_{ij}^{min}$ ,  $P_{ij}^{max}$ ]
5. Entering the loss parameters of transmission lines, which are in the form of B matrix
6. Finding the maximum point in the load curve
7. Subtracting the *storestage* value of the minimum production from the maximum point
8. Re-finding the new maximum
9. Re-subtracting the *storestage* value of the minimum production from the new maximum
10. Checking the requirements of achieving the maximum production at any time step
11. Finding the minimum point in the load curve
12. Adding the *storestage* value of the minimum production to the minimum point
13. Re-finding the new minimum
14. Re-adding the *storestage* value of the minimum production to the new minimum
15. Checking the condition of achieving the maximum production at any time step
16. Checking the condition of the capacity stored in the upper tank
17. Iteration until condition 14 is violated
18. Choosing the total 24-hour  $G_i[n]$
19. Minimizing the thermal power plant costs using *PSO* algorithm
20. Specifying the values of  $PHp[n]$ ,  $PHg[n]$ , and  $P_{Gi}$  for each thermal unit
21. Calculating the maximum profit of utilizing storage-pump power plant

The above solution procedure results in optimal planning and economical distribution of storage-pump power plants and thermal power plant units. It aims to minimize the total cost. Ultimately, given the optimal planning, the maximum economical profit gained by utilizing storage-pump power plant will be specified.

#### 4. Determining the most optimal storage capacity considering system conditions

In previous sections, by optimal planning for storage-pump power plants to maximize the profit in a one-day period on one hand, and minimizing the cost of thermal power plants to determine the amount of their production based on the storage capacity on the other hand, optimization was performed using *PSO* algorithm.

In this condition, the upper tank capacity and the amount of stored energy will be selected as a constant value as input, and the total daily cost of thermal power plant production will be calculated using Equation (19) in presence of storage-pump power plant, and using Equation (20) in absence of storage-pump power plant. The difference between these two values is the profit of utilizing storage-pump power plant, which is given in Equation (21).

$$IC_{\_withPHS} = F_{T1}(P) = \sum_{i=1}^N F(P_{Gi}) \quad (19)$$

$$IC_{\_withoutPHS} = F_{T2}(P) = \sum_{i=1}^N F(P_{Gi}) \quad (20)$$

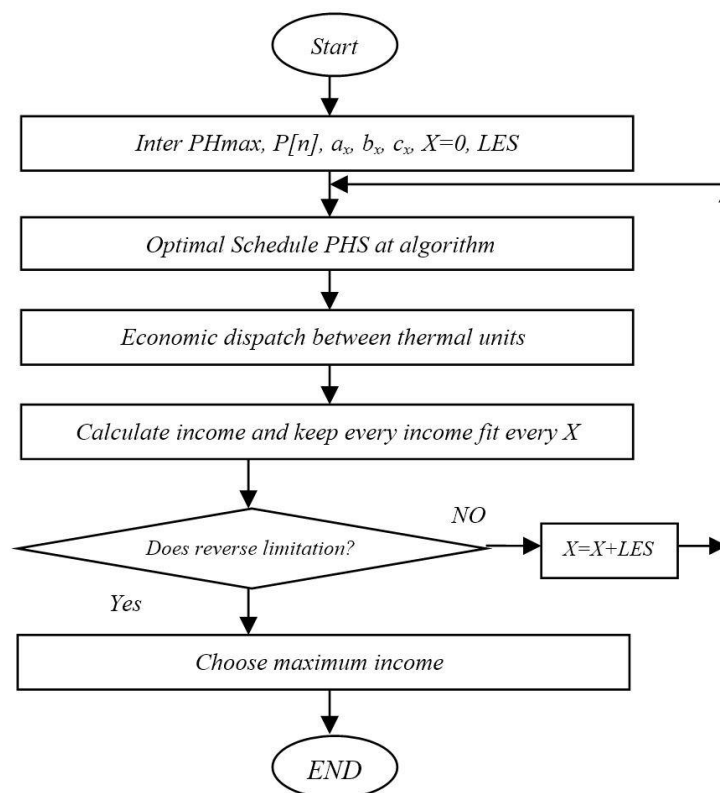
$$IC_{incom} = IC_{\_withoutPHS} - IC_{\_withPHS} \quad (21)$$

$IC_{incom}$  varies with different storage capacities. Changing storage capacity results in changing the planning for a storage-pump power plant, and various planning will naturally result in changing the optimal arrangement of thermal power plants. The total cost will also vary with changing each thermal unit production procedure. Changing thermal costs will change  $IC_{incom}$  as a result.

In order to determine the optimal storage capacity, the capacity must be specified in a way that maximizes  $IC_{incom}$  based on the system conditions.

Optimization with direct method must be used to maximize and optimize  $IC_{incom}$ .

In optimization by stochastic algorithms, since the planning and cost minimization calculations are carried out by stochastic algorithms themselves, the computation time becomes too much. Therefore, the proposed method to optimize  $IC_{incom}$  is done according to Figure (4).



**Figure 4. Optimal production algorithm of choosing  $PHp[n]$  and  $G[n]$**

The parameter  $LES^4$  is the input to the algorithm and show the increased amount of the stored energy in each iteration. As  $LES$  value becomes smaller, the more iterations will be required and the computation time gets higher, but the calculations get closer to the optimal point. In each iteration of the above algorithm, the PSO program runs for the desired tank capacity. Given the tank capacity and the considered amount of storage energy, the optimal plan of storage-pump power plant runs, and the production costs of thermal power plants will become minimized and the production arrangement of each thermal power plant will be determined based on the performed planning. The profit will also be determined in that storage capacity.

In Figure (4), the profit is calculated for different storage capacities and finally, the storage capacity at which the profit reaches the maximum value is determined. Given the network conditions and thermal power plants, the storage energy of storage-pump power plant yielding the maximum economic profit will be estimated. Based on these calculations, it will be determined that what type of storage-pump power plant with how much storage capacity has to be established, given the network parameters, in order to achieve the maximum economic profit.

### 5. The studied system [38]

The studied system is a 26-bus system with 6 thermal units with the characteristics given below. The cost parameters of each thermal power plant and the approximate production of each power plant is given in Table (2).

**Table 2. Characteristics and parameters of thermal units**

Unit	$a_x$	$b_x$	$c_x$	$PG_{,x} \text{ min}$	$PG_{,x} \text{ max}$
1	0.070	7	240	100	500
2	0.095	10	200	50	200
3	0.090	8.5	220	80	300
4	0.090	11	200	50	150
5	0.080	10.5	220	50	200
6	0.075	12	190	50	120

<sup>4</sup> Length power step

The hourly load of 24 hours of the system in terms of demands is according to Table (3).

**Table 3. Hourly load of the studied system**

Time (H)	Load (MW)	Time (H)	Load (MW)	Time (H)	Load (MW)	Time (H)	Load (MW)
1	955	7	989	13	1190	19	1159
2	942	8	1023	14	1251	20	1092
3	935	9	1126	15	1263	21	1023
4	930	10	1150	16	1250	22	984
5	935	11	1201	17	1221	23	975
6	963	12	1235	18	1202	24	960

Based on loss modeling in the study, matrix B of the above system is in the following form. The adjusted parameters in PSO optimization algorithm are given in Table (4).

$$B = 10^{-3} * \begin{bmatrix} 1.7 & 1.2 & 0.7 & -0.1 & -0.5 & -0.2 \\ 1.2 & 1.4 & 0.9 & 0.1 & -0.6 & 0.1 \\ 0.7 & 0.9 & 3.1 & 0.0 & -0.10 & -0.6 \\ -0.1 & 0.1 & 0.0 & 0.24 & -0.6 & -0.8 \\ -0.5 & -0.6 & -0.1 & -0.6 & 12.9 & -0.2 \\ -0.2 & -0.1 & -0.6 & -0.8 & -0.2 & 15.0 \end{bmatrix}$$

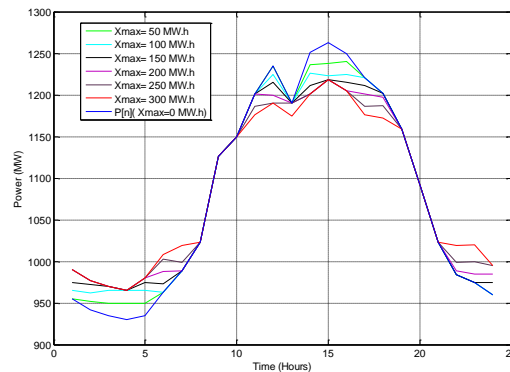
**Table 4. PSO algorithm input parameters**

Maximum Number of Iterations	1000	MaxIt
Population Size (Swarm Size)	100	nPop
Inertia Weight	1	w
Inertia Weight Damping Ratio	0.99	wdamp
Personal Learning Coefficient	2	c1
Global Learning Coefficient	2	c2

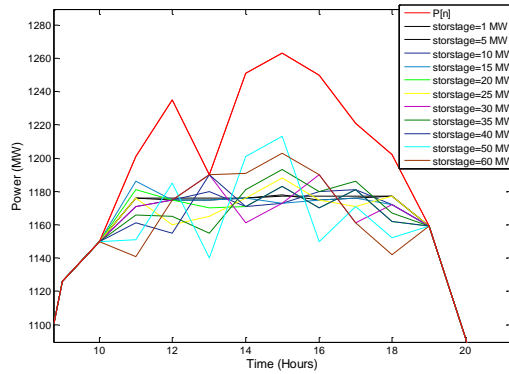
## 6. Simulation results

After solving the optimization problem by storage-pump power plant planning for a certain tank capacity and different maximum power of storage-pump power plants,  $LES=25MW.h$  or in the form of Figure (5) or (7).

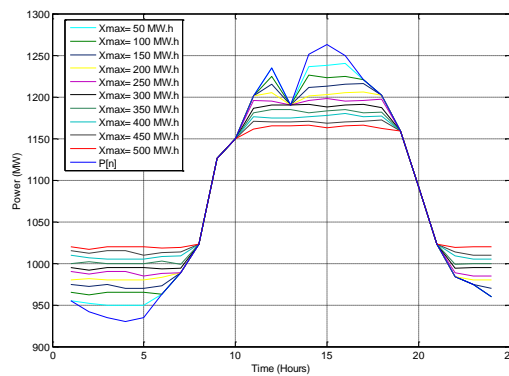
Figures (6) and Figure (11) show the thermal production curve in a given storage capacity and the constant maximum production of storage-pump power plant, but for different *storstage* values at peak load. Figure (6) shows the thermal production curve in a given storage capacity and the constant maximum production of storage-pump power plant, but for different *storstage* values at peak load.



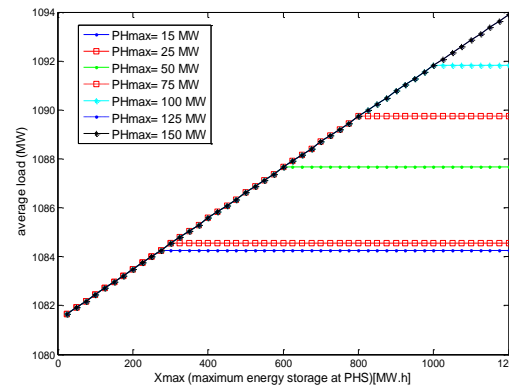
**Figure 5. Plot of cumulative thermal production,  $LES=50MW.h$ ,  $PH_{max}=50MW$ ,  $storstage=5MW$**



**Figure 6. Thermal production of system power plants at peak load for  $X= 400 \text{ MW.h}$ ,  $PH_{max}= 100 \text{ MW}$ , and different storstage values**



**Figure 7. Plot of cumulative thermal production,  $LES=50\text{MW.h}$ ,  $PH_{max}=125\text{MW}$ , storstage=5MW**



**Figure 8. Average load versus storage capacity at different values of  $PH_{max}$**

As the storage capacity of storage-pump power plants and the maximum production of thermal power plants increases, the load curve becomes more flat and the load factor increases. Figure (9) shows the load factor versus storage capacity for different values of  $PH_{max}$ .

The amount of stored energy at different hours of day in the upper tank of storage-pump power plant for different capacities is shown in Figure (11). Production and consumption of storage-pump power plants for different storage capacities is shown in Figure (12). Increasing the average load with storage capacity at different values of  $PH_{max}$  is shown in Figure (8).

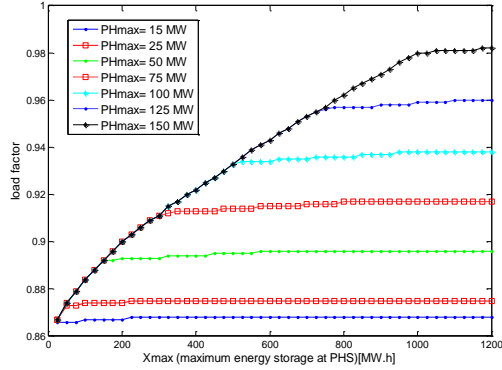


Figure 9. Load factor versus tank capacity at different values of  $PH_{max}$

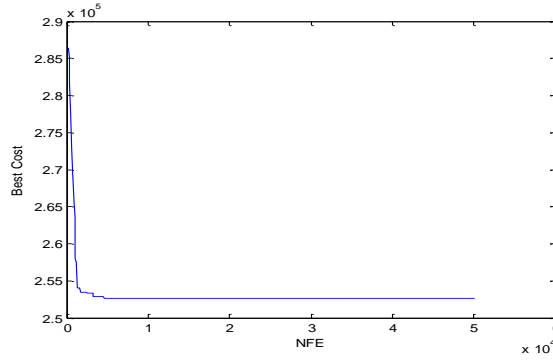


Figure 10. Convergence of the calculated cost versus the number of iterations in PSO

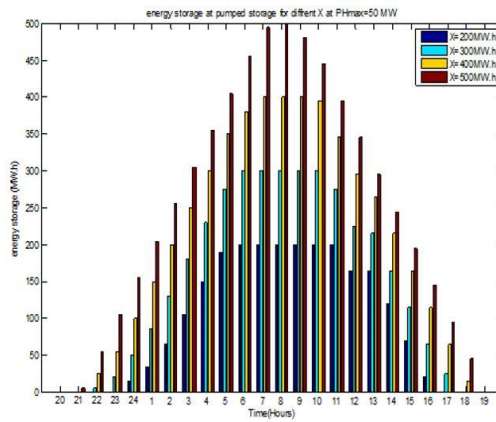
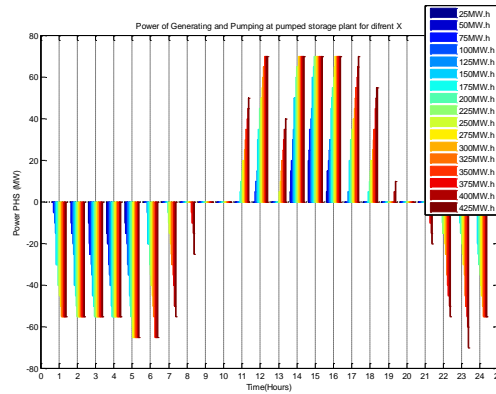
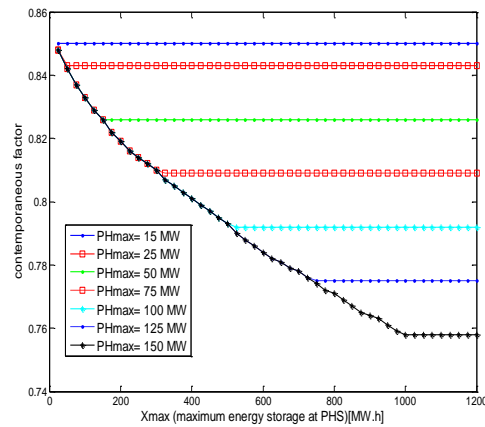


Figure 11. The amount of stored energy at different hours of day in storage-pump power plant versus different capacities

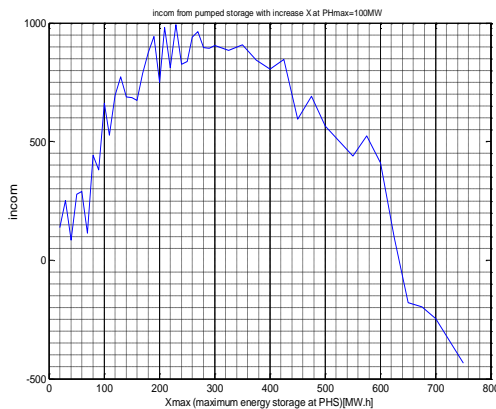


**Figure 12. Production and consumption of storage-pump power plant versus different storage capacities**

Figure (10) shows the convergence of cost in PSO algorithm. The design must be in a way that increases the efficiency as much as possible, so that the changes of average load reduces or becomes fixed. The studied system has 6 power plant units. Figure (13) shows the changes of reserve coefficient parameter versus storage capacity. Figure (14) shows decreasing the contemporaneous factor at different values of  $PH_{max}$ .



**Figure 13. Contemporaneous factor versus storage capacity at different values of  $PH_{max}$**



**Figure 14. Storage-pump power plant profit versus storage capacity at  $PH_{max}=100$  MW**

The amount energy stored in the upper tank at different hours of they is shown in the Figure (13). According to this plot, as the tank capacity increases, the time at which the energy is stored at full capacity

decreases. For example, the energy is stored in full capacity for 6 hours in the blue curve, while this time in the red curve is reduced to 2 hours. The income and earnings of a storage-pump power plant varies with different amounts of stored energy, which is shown in Figure (14). As it can be seen in this figure, income versus capacity is bell shaped, i.e., increasing the stored energy increases  $IC_{incom}$  up to certain level, after which  $IC_{incom}$  decreases with increasing the stored energy. This figure can be used to estimate the most optimal storage capacity. Since the  $IC_{incom}$  curve is bell shaped at low tank capacities, and increasing the capacity would naturally increase use of the storage-pump power plant, the profit will increase. But after a certain value, because of the efficiency of storage-pump power plant on one hand, and flattening of load curve of storage-pump power plant due to direct relationship between the cost of thermal power plant and network load on the other hand, approaching curve peaks and pits reduces the flexibility of thermal power plants output, which will result in decreasing  $IC_{incom}$  because of efficiency of storage-pump power plant.

It is obvious from the figures that there is a direct relationship between the parameters  $IC_{incom}$  and  $PH_{max}$ .  $IC_{incom}$  increases with  $PH_{max}$  of storage-pump power plant. As higher values are selected for  $PH_{max}$  in designing, the income and profit will be higher. It was seen in the studied system that the highest profit of storage-pump power plant was obtained at  $X=225 MW.h$  storage capacity. Therefore,  $X=225 MW.h$  is suggested as the storage capacity of the studied system to have the highest profitability at this condition.

The production distribution of thermal units in presence of a storage-pump power plant with  $X_{max}=300 MW.h$ ,  $PH_{max}=100 MW$ , and  $storestage=1 MW$  is given in the following table.

Table (5) shows the production planning and economical distribution of production and consumption in 24 hours of thermal power plants. The cost of thermal power plants in presence/absence of storage-pump power plant is also determined at each hour, as well as the economic profit and loss at each hour, resulted from using storage-pump power plant.

**Table 5. Distribution of thermal and storage-pump power plants and cost at each hour (all powers are in MW)**

Time (Hours)	Genrator1 (MW)	Genrator2 (MW)	Genrator3 (MW)	Genrator4 (MW)	Genrator5 (MW)	Genrator6 (MW)	Sum cost With PHS	Sum cost Without PHS	Income (\$)	PH[n] (MW)	Losses (MW)
1	119.8942	174.6062	147.3681	171.2533	152.6709	229.2069	24752	23227	-1525	-40	8.255
2	119.9866	182.4035	134.5435	170.6855	161.0298	226.3722	24761	22745	-2015	-53	8.489
3	119.9994	173.9812	146.2648	163.966	153.964	236.8295	24751	22489	-2261	-60	8.331
4	119.9759	180.4291	149.8949	165.0068	151.1084	227.5849	24715	22310	-2405	-64	8.09
5	119.9986	174.9103	140.3034	178.4572	150.509	229.8312	24714	22487	-2227	-59	8.137
6	119.9114	175.5761	146.5303	170.7947	231.4855	231.4855	24710	23527	-1183	-31	8.339
7	119.995	178.1159	129.9556	179.4035	156.4757	230.0548	24711	24518	-192	-5	8.264
8	120	178.7521	150	171.0919	157.2621	245.8913	25856	25856	0	0	8.853
9	119.9707	200	149.9998	196.7309	179.1287	280.1699	30206	30206	0	0	10.671
10	120	198.373	149.9982	209.8567	189.4465	282.3199	31298	31298	0	0	11.027
11	120	199.9423	149.9944	220.2191	199.9976	298.8485	33141	33730	588	12	11.664
12	119.9708	199.4235	150	225.6555	200	293.9495	33142	35457	2315	46	11.671
13	119.9999	199.8987	149.9998	222.4912	192.5008	304.1095	33144	33192	48	1	11.648
14	119.9524	199.9948	149.99	226.6523	195.4128	296.9964	33141	36301	3160	62	11.664
15	119.9985	199.9988	149.9149	221.1625	197.2819	300.6434	33143	36951	3807	74	11.68
16	120	199.9924	150	219.4591	195.6319	303.9176	33142	36256	3114	61	11.658
17	120	199.2602	150	227.0509	199.9975	292.6914	33141	34735	1593	32	11.662
18	119.9661	199.9621	149.7173	226.1858	195.1308	299.0379	33191	33780	588	12	11.689
19	120	199.9927	149.6953	209.3146	195.6436	284.3538	31717	31717	0.000	0	11.188
20	119.778	199.1484	150	192.8624	169.4254	260.784	28713	28713	0	0	10.195
21	119.9752	187.1873	149.9882	171.5743	158.9824	235.2921	25853	25853	.0	0	8.845
22	119.9661	172.5678	142.259	173.1051	158.2734	227.8458	24714	24326	-388	-10	8.399
23	117.6134	178.1503	132.4725	169.5385	159.7786	236.4359	24717	23987	-729	-19	8.325
24	119.9989	177.5216	140.7321	172.6566	153.4167	229.6767	24721	23419	-1304	-34	8.314

### 6. Conclusion

Utilizing storage-pump power plant in a network whose load factor is small, the difference between the peak and pit of its daily load curve is high, and there is a significant difference between the production cost



of the units at peak and the base production cost of the units is more profitable. As the tank capacity and the maximum production and consumption power of a storage-pump power plant increases, the improvement in technical parameters will be more evident. There are two objective functions, one for maximizing the storage-pump power plant profit, considering all constraints associated with the storage-pump power plant, the other for minimizing storage-pump power plant cost, considering the cost functions and the constraints on minimum and maximum production from a thermal power plant equivalent to the storage-pump power plant. The two objective functions in a problem are consistent and complement each other, and are optimized simultaneously. The objective function to maximize the profit results in optimal planning of storage-pump power plant, so that solving this function would yield to goal of achieving the best time of entry/exit into operation and the amount of production and consumption at any time in a storage-pump power plant. Given the objectives of a storage-pump power plant, which are *reducing* the cost of thermal power plants and the transmission losses between them, the arrangement of thermal power plants will be specified according to the cost of each unit by the optimal planning carried out. The operation procedure of each thermal unit will also be specified based on the most optimal planning carried out for the storage-pump power plants, using PSO optimization algorithm. In the proposed method, there is a 24-hour economical profit, given a certain tank capacity and optimal planning for this amount of storage. It is the maximum profit of a storage-pump power plant, obtained by presenting a proposal regarding the sizing and estimating the tank capacity and the optimal amount of storage. Tank capacity is not constant, but is selected as an unknown value, and the maximum profit in the system will be calculated at different storage capacities, using previous results. Choosing different storage capacities and calculating the maximum profit using previous results also raises an optimization problem. The profit must be calculated based on the environmental conditions, the conditions prevailing in the power system, and the maximum hourly load of that power system at different capacities. Then, the highest value of these maximum profits will be selected, and the storage capacity associated with it will be determined, which shows the maximum profit is obtained at this storage capacity, and this amount of storage capacity is an estimate of sizing of a storage-pump power plant in its associated power system.

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