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Formulation of the density matrix in v-type three-level atom

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ABSTRACT
In this paper, the density matrix equations for a v-type three-level system are introduced; the relationship between the two types of decay rates, the spontaneous decay rate of the net population of levels and the decay rate of the dipole moments are explained here. The phenomenological part of the equations resulting from these decays is discussed. These concepts can be generalized to four- or more-level atoms.
Keywords: density matrix, atom, level

Introduction
The most important tool in the assessment of phenomena such as bipolarity, electromagnetically induced transparency (EIT), and lasing without inversion [1], or writing fundamental equations of Bloch for two-, three-, or four-level atoms [2], is the formulation of the density matrix. The core of this set of equations is the Liouville equation, which deals with the time evolution of the net population of equilibria and the population of dipoles resulting from permitted and forbidden transitions, but each of these equations consists of two main parts: the first part includes terms resulting from the interaction of the electromagnetic field with an atomic system that is obtained directly by the Liouville equation of motion [3], and the second part includes terms that result from the spontaneous decay of the levels with a rate of $\gamma_{\circ}$ and the decay of dipole moments with a rate of γ_{ij}. These terms, which are the effect of quantum interference, are often added to the density matrix equations phenomenologically. We introduce this second part of the equation while introducing a v-type three-level atomic system.

Three-level atomic system and density matrix
Consider an electron at a lower level $|b\rangle$ with the wave function of ψ_b; the expected value of the position of this electron is constant until the transition occurs and the electron does not radiate. But when the electron is excited to the higher level of $|a\rangle$, the position of the electron fluctuates with the frequency of $\omega = \frac{E_a - E_b}{h}$, and the expected value $\langle x \rangle$ is no longer constant. The general condition required for an atom to radiate in an excited state is that the integral $\int_{-\infty}^{\infty} x\psi_b\psi_a^* dx$ is not zero. Because the intensity of radiation is proportional to this integral. Transitions for which this integral is finite are called permitted transitions, while if the value of this integral is zero, that transition is called a forbidden transition.

Consider a v-type three-level closed-loop atomic system as shown in Figure 1 [4]. In this atom, the two higher sublevels $|1\rangle$ and $|2\rangle$ with the base level $|3\rangle$ are paired by a single-mode laser field with amplitude ε and angular frequency ω. The resonant frequencies between the higher levels $|1\rangle$ and $|2\rangle$ and the base level $|3\rangle$ are ω_{13} and ω_{13}, respectively.

ω_{12} is also the frequency difference between the excited levels, i.e. $\omega_{13} - \omega_{23} = \omega_{12}$

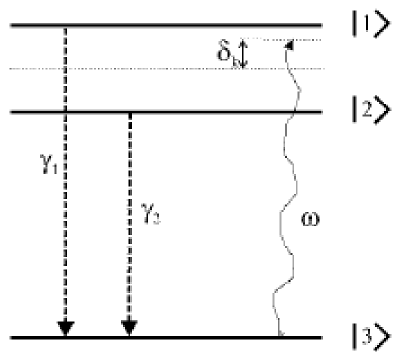

Figure 1

The density matrix equations for the mentioned system are as follows:

$$\frac{\partial \rho_{11}}{\partial t} = -\gamma_1\rho_{11} - \gamma_{12}(\rho_{12} + \rho_{21}) + i\Omega_1\rho_{31} - i\Omega_1^*\rho_{13} \quad , \tag{1}$$

$$\frac{\partial \rho_{22}}{\partial t} = -\gamma_2\rho_{22} - \gamma_{12}(\rho_{12} + \rho_{21}) + i\,\Omega_2\rho_{32} - i\Omega_2^*\rho_{23} \quad , \tag{2}$$

$$\frac{\partial \rho_{13}}{\partial t} = -\Gamma_{13}\rho_{13} - \gamma_{12}\rho_{23} - i\Omega_1(\rho_{11} - \rho_{33}) - i\Omega_2\rho_{12} \quad , \tag{3}$$

$$\frac{\partial \rho_{23}}{\partial t} = -\Gamma_{23}\rho_{23} - \gamma_{12}\rho_{13} + i\Omega_2(\rho_{33} - \rho_{22}) - i\Omega_1\rho_{21} \quad , \tag{4}$$

$$\frac{\partial \rho_{12}}{\partial t} = -\Gamma_{12}\rho_{12} - \gamma_{12}(\rho_{11} + \rho_{22}) + i\Omega_1\rho_{32} - i\Omega_2^*\rho_{13} \quad , \tag{5}$$

$$\Gamma_{23} = \left(\frac{\gamma_2}{2} + i\Delta_2\right) \quad , \tag{6}$$

$$\Gamma_{13} = \left(\frac{\gamma_1}{2} + i\Delta_1\right) \quad , \tag{7}$$

$$\Gamma_{12} = \left[\frac{(\gamma_1 + \gamma_2)}{2} + i\omega_{12}\right] \quad , \tag{8}$$

The last two terms of equations (1) to (5) of the Liouville equation of motion are obtained by considering the Hamiltonian disorder as $H1 = \mathrm{e.x.E(x)}$ in the interaction image. But how are equations (6), (7), and (8) and the first two terms of each of equations (1) to (5) obtained? First, in order to add the decay of spontaneous radiation-dependent atomic levels, we consider two categories of decay; one is the decay of the net population of equilibria with the decay constant $\gamma_{\circ}$ due to spontaneous radiation, and the other is the decay of electrical dipoles with the decay constant γ_{ij}. These decay terms, which are dependent on quantum interference, are often added to the density matrix equations as phenomenological phenomena that are the effect of quantum interference. To understand the relationship between these two types of decay, consider the following:

Consider two levels $|a\rangle$ and $|b\rangle$ so that $|a\rangle$ is the higher level and $|b\rangle$ is the lower level; the dipole moment between $|a\rangle$ and $|b\rangle$ has a population twice the population of $|b\rangle$ alone. According to Figure 2, we assume that first the base level $|b\rangle$ is completely full and $|a\rangle$ is completely empty, that is, we consider the electrons of level $|b\rangle$ with their corresponding holes in $|a\rangle$. By starting the pump process, which is done by a laser electromagnetic field, an ensemble of electrons transitions from $|b\rangle$ to $|a\rangle$, resulting in their corresponding holes in $|b\rangle$, and because the transition process is accompanied by spontaneous irradiation of electrons, we can consider the approximation that in equilibrium the number of electrons in each level is equal (for example, we have N electrons in each level and therefore 2N dipole moment with opposite directions between the two levels). This means that in the decay of bipolar moments, half of them are destroyed by excitation from $|b\rangle$ to $|a\rangle$ and the other half are destroyed by emission from $|a\rangle$ to $|b\rangle$. However, population of the level $|b\rangle$ is only the result of the excitation process and the decay of the population $|a\rangle$ is only the result of the emission process. So the bipolar moments in the decay process are half the share of the decay of the net population at the level $|b\rangle$, which is dependent on spontaneous radiation. For example, according to equation (7), we have the following for the decay rate of bipolar moments: $\Gamma_{13} = \frac{\gamma_1}{2} + \cdots$. Note that in the decay process of bipolar moment, when it decays, its direction changes in the opposite direction, so it is no longer the previous bipolar moment, although another has been created.

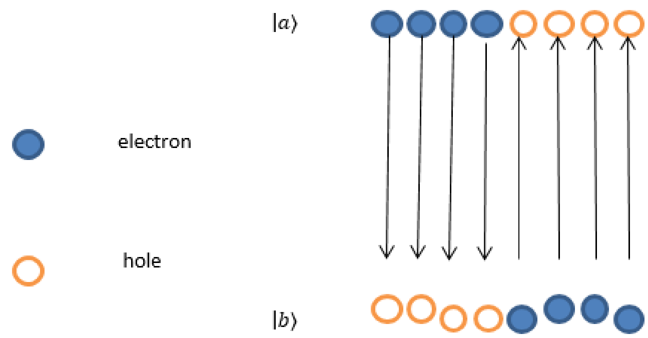

Figure 2

The complete term of Γ_{13} in equation (7) is: $\Gamma_{13} = \frac{\gamma_1}{2} + i\Delta_1$. To write the second term of the equation, we consider that when the population of electrons is pumped by a laser electromagnetic field from a lower level to a higher level, if $\omega_{ab} = \omega_a - \omega_b$ is equal to the frequency of the laser light, ω, the resonant effect causes the electrons to be precisely excited to level $|a\rangle$. But if $\omega < \omega_{ab}$, or in other words $\Delta = \omega_{ab} - \omega$ and $\Delta \neq 0$, so that Δ is called mistuned laser field, then the electrons are excited to a level slightly lower than $|a\rangle$ level; as a result, they are weak and tend to radiate. So in the calculation of $\dot{\rho_{ab}}$, factor Δ represents the weakness of these electrons, which is an interfering factor in the decay process.

As the nature of Δ is frequency and inverse of time, therefore, it has the nature of the decay rate and has the dimension T^{-1}. Also, this share of decay, which is dependent on mistune, has a phase difference of $\frac{\pi}{2}$ from the share of γ_i, which is dependent on spontaneous radiation. Clearly, the higher the pumping rate, the more effective will be Δ from the spontaneous radiation, resulting in a decrease in the share of spontaneous radiation. so this process of inverse change presents Γ_{13} as a complex quantity of which the imaginary part is Δ. For example: $\Gamma_{13} = \frac{\gamma_1}{2} + i\Delta_1$ or $\Gamma_{23} = \frac{\gamma_2}{2} + i\Delta_1$.

In Equation (8), the first term $(\frac{\gamma_1+\gamma_2}{2})$ states that the change in the population of the moment μ_{12} depends on the change in the population of the moments μ_{13} as well as μ_{23}. To write the second term of Equation (8), we explain the concept of coherence. The main non-diameter elements of the density matrix represent the atomic polarization or moment population of the system's electrical dipoles, which is a measure of quantum coherence between levels. Coherence means the transition between two states achieved by coherent overlap between them. For an atom of two levels, the wave function is written as follows: $|\Psi(t)\rangle = C_a(t)|a\rangle + C_b(t)|b\rangle$ where C_a and C_b are the amplitude of the atom in the states $|a\rangle$ and $|b\rangle$.

C_a and C_b are the slopes of the slow-wave amplitude function: $C_a = c_a e^{-\omega_a t}$ and $C_b = c_b e^{-\omega_b t}$. The coherence between states $|a\rangle$ and $|b\rangle$ can be described as $c_a^* c_b e^{-i(\omega_b - \omega_a)}$. If the phase relation $(\omega_b - \omega_a)$ t = constant survives, coherence is obtained between the two states [4]. If the level $|a\rangle$ has a wave function with a number of ω_a peaks per time unit and the level $|b\rangle$ has a wave function with a number of ω_b peaks per time unit, after the interaction of these two waves we will have a number of $\omega_b - \omega_a$ peaks per time unit (this is reminiscent of the phenomenon of pulsation between two waves in the classical mode), which is equal to the rate of transition from $|b\rangle$ to $|a\rangle$.

This is why, if the laser wave with the Δ mistune causes a transition and weak level lower than the original level, the electrons will decay at the same rate as they were excited. Now in this three-level system, instead of Δ, $\omega_{12} = \omega_1 - \omega_2$, is responsible for the decay rate between $|1\rangle$ and $|2\rangle$, because the laser field is not applied between the two levels as the transition $|1\rangle \rightarrow |2\rangle$ is forbidden. The dipole moments μ_{12} and μ_{21} are unstable, and the electron excited from $|2\rangle$ to $|1\rangle$ eventually decays to $|3\rangle$, or the electron emitted from $|1\rangle$ to $|2\rangle$ eventually decays to $|3\rangle$. When the transitions $|1\rangle$ $|3\rangle$ and $|2\rangle$ $|3\rangle$ are not effective, the term $i\omega_{12}$ will be effective. Because the moments μ_{21} and μ_{12} are not stable, they decay rapidly at the same rate at which they were excited (at the rate ω_{12}). It is clear that if the transitions $|1\rangle$ to $|3\rangle$ and $|2\rangle$ to $|3\rangle$ are effective, the term $i\omega_{12}$ will not be effective, then this term will have a phase difference of $\frac{\pi}{2}$ with the first sentence of Equation (8), i.e. $\frac{\gamma_1+\gamma_2}{2}$. But how the second term in Equations (1) to (5) are obtained:

We start with Equation (3). $\dot{\rho_{13}}$is the temporal variation of the transition polarity $|1\rangle$ to $|3\rangle$ or in other words, it represents the population changes of μ_{13} moment. We can assume that if an electron in the level $|1\rangle$ is transferred to the level $|3\rangle$ it can move in the path $|1\rangle$ to $|2\rangle$ to $|3\rangle$, i.e. it may first move to the level $|2\rangle$. This electron, which goes to $|2\rangle$ with a rate of γ_{12}, affects the population of μ_{23} moments by multiplying γ_{12} by ρ_{23} with a negative sign, because this electron eventually will reduce the population of μ_{13} moments by passing from $|1\rangle$ to $|3\rangle$. The same reasons can be applied in the second term of Equation (4) in which the electron is transferred along the path $|2\rangle$ to $|1\rangle$ to $|3\rangle$. In Equation (1) the level $|1\rangle$ population decreases under two processes: one is decay of $|1\rangle$ to $|3\rangle$ with the decay rate of γ_1 calculated in the first sentence and the other is that the level $|1\rangle$ population changes depending on the decay of the dipole moments μ_{12} and μ_{21}, i.e. the electron in the level $|1\rangle$ falls to its hole at $|2\rangle$ or vice versa. Since the transitions of $|1\rangle$ $|2\rangle$ are forbidden, the result is that an electron that has gone from $|1\rangle$ to $|2\rangle$ or vice versa will eventually decay to $|3\rangle$. Therefore $\dot{\rho}_{11}$ will be affected by the decrease in population of inductive moments μ_{12} and μ_{21}, i.e. for the second part of change of population ρ_{11} we have: $-\gamma_{12}(\rho_{12} + \rho_{21})$ and the same reasons are true for the second term of Equation (2), $\dot{\rho}_{22}$. Type equation here. But in Equation (5), the population change of the moments μ_{12} is affected by the population change of the levels $|1\rangle$ and $|2\rangle$ with the decay rate γ_{12}. Note that to change the permitted population of transitions, the path is two-channel and that path contains two excitations, or two emissions, or one excitation and one emission, so that a channel must contain a forbidden transition (excitation or emission) that ultimately results in a permitted transition. Now we can not introduce this two-channel path to change the population of the forbidden transitions μ_{12} and μ_{21}. For example, to form the moment μ_{12}, the path $|1\rangle$ to $|3\rangle$ to $|2\rangle$ is not possible because both channels are permitted transitions that are independent and have no effect on each other, so the change in the population of the forbidden moments is effective from the change in the population of the levels, albeit with the a rate of γ_{12}. This means that the change in the population of forbidden moments, $\dot{\rho}_{12}$, Is directly effective from the decay rate Γ_{12}. γ_{12}, which we call the forbidden moment decay constant, has an indirect contribution to $\dot{\rho_{12}}$. Note that even if a term in the form $\gamma_{12}.\rho_{12}$ is present in Equation (5), this expression is almost zero because both γ_{12} and ρ_{12} are very small quantities.

Relationship between decay constant γ_{12} and decays γ_1 and γ_2

The relationship between the decay constant of dipole moments, γ_{12}, with the decay of the net population of the levels γ_1 and γ_2 is obtained from the following relation [5]:

$$\gamma_{12} = \gamma_{21} = \pi \sum_{ks} [\overrightarrow{\mu_{13}} . \overrightarrow{g_{ks}}(r)] . \left[\overrightarrow{\mu_{23}^*} . \overrightarrow{g_{ks}^*}(r)\right] . \delta^3(k - k_0)$$

So that $k_0 = \frac{(k_1 + k_2)}{2}$ and $k_1 = \frac{\omega_1}{c}$ and $k_2 = \frac{\omega_2}{c}$.

k is the vector of a radiant photon, s is its polarity, and $\overrightarrow{g_{ks}(r)}$ is the mode function of multi-mode 3D vacuum field written as:

$$(r) = \left(\frac{ck}{2\pi\varepsilon 0 h(2\pi)^3}\right)^{\frac{1}{2}} \widehat{e_{ks}} \,.\, e^{ik.r}\overrightarrow{g_{ks}}$$

Which is evaluated in position r of radiative dipole moment. For vacuum modes we use the flat wav (r) $\overrightarrow{g_{ks}}$. The unit vectors $\widehat{e_{k1}}$ and $\widehat{e_{k2}}$ are orthonormal so $\overrightarrow{g_{k1}}$ is perpendicular to $\overrightarrow{g_{k2}}$.

According to what we have said before, the spontaneous decay rate is twice that of the bipolar moment decay rate, i.e. we have:

$$\gamma_1 = 2\gamma_{11} = 2\pi\left[(\overrightarrow{\mu_{13}}\,.\,\overrightarrow{g_{k1}})\left(\overrightarrow{\mu_{13}^*}\,.\,\overrightarrow{g_{k1}^*}\right) + (\overrightarrow{\mu_{13}}\,.\,\overrightarrow{g_{k2}})\left(\overrightarrow{\mu_{13}^*}\,.\,\overrightarrow{g_{k2}^*}\right)\right] \qquad \text{(for } K = K_0\text{)}$$

$$\gamma_{12} = \pi\left[(\overrightarrow{\mu_{13}}.\overrightarrow{g_{k1}}).\left(\overrightarrow{\mu_{23}^*}.\overrightarrow{g_{k1}^*}\right) + (\overrightarrow{\mu_{13}}.\overrightarrow{g_{k2}}).\left(\overrightarrow{\mu_{23}^*}.\overrightarrow{g_{k2}^*}\right)\right]$$

And also:

The angles between the dipole moments and $\overrightarrow{g_{k1}}$ and $\overrightarrow{g_{k2}}$ are shown in Figure (3). By scalar multiplying in the above relations we will have:

$$\gamma_1 = 2\pi[(\mu_{13.}\cos\alpha).(\mu_{13}^*.\cos\alpha) + (\mu_{13.}\sin\alpha).(\mu_{13}^*\sin\alpha)]$$

$$\gamma_1 = 2\pi|\mu_{13}|^2 \quad , \qquad (9)$$

$$\gamma_2 = 2\pi|\mu_{23}|^2 \quad , \qquad (10)$$

$$\gamma_{12} = \pi[(\mu_{13}.\cos\alpha).(\mu_{23}^*.\cos\varphi) - (\mu_{13}.\sin\alpha).(\mu_{23}^*.\sin\varphi)] \quad ,$$

$$\gamma_{12} = \pi|\mu_{13}|.|\mu_{23}^*|.[\cos\alpha\,.\cos\varphi - \sin\alpha\,.\sin\varphi] = \pi|\mu_{13}|.|\mu_{23}|.\cos(\alpha+\varphi) \quad . \qquad (11)$$

We put: $\theta = \alpha + \varphi$, where θ is the angle between $\overrightarrow{\mu_{13}}$ and $\overrightarrow{\mu_{23}}$:

$$\cos\theta = \frac{\overrightarrow{\mu_{13}}.\overrightarrow{\mu_{23}}}{|\overrightarrow{\mu_{13}}|.|\overrightarrow{\mu_{23}}|}$$

Then, by comparing equations (9) and (10) with equation (11), we obtain the final result:

$$\gamma_{12} = \frac{\sqrt{\gamma_1.\gamma_2}}{2}\left(\frac{\overrightarrow{\mu_{13}}.\overrightarrow{\mu_{23}}}{|\overrightarrow{\mu_{13}}|.|\overrightarrow{\mu_{23}}|}\right) \quad . \qquad (12)$$

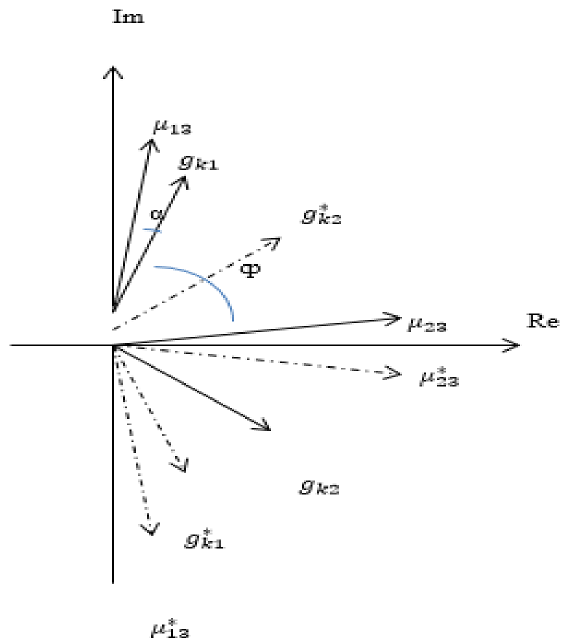

Figure 3

Conclusion

In this paper, we discuss the phenomenological part of the density matrix equations that result from decay. According to what has been said, the rate of spontaneous decay is twice that of the rate of decay of bipolar moments, and the decay path to change the population of permitted bipolar moments is two-channel, so that one channel must belong to the forbidden transition (excitation or transition). But to change the population of forbidden bipolar moments, we can not introduce such a two-channel path. Finally, we have explained the relationship between the dipole moment decay constant γ_{ij} with γ_i and γ_j. This relationship is in the form of Equation (12).

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