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Resource allocation in a centralized system using inverse data envelopment analysis

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ABSTRACT

One of the most important and practical branches of research in action is known as data envelopment analysis. Data envelopment analysis is a method to evaluate the relative efficiency of a decision-making unit compared to other units that makes use of the same inputs to produce the same outputs. One of the most widely used issues in data envelopment analysis is changing of one index and determination of other indicators in order to maintain or improve efficiency. This is the called inverse data envelopment analysis. In today's world, given the advancement of technologies, we are witnessing states where all decision-making units are supervised by a central unit that by itself monitors the efficiency of other units. In these centralized systems, we sometimes encounter situations where we have to increase or decrease the output due to internal (such as insufficient labor force, employee motivation, change in managerial policy, etc.) or external pressures (such as sanctions, exports, etc.) on the system. These factors certainly affect the consumption level of inputs. In the present study, it is examined to what degree these changes affect the inputs of the entire system and how and to what extent this possible change in the level of inputs will be distributed among the decision-making units in the centralized system such that the maximum efficiency is produced not only for each unit, but also for the entire centralized system.In this paper, resource allocation was done in a centralized system using inverse data envelopment analysis. The proposed inverse data envelopment analysis model is in fact an SBM or additive analysis model. The model was solved by a numerical example using GAMS software. A numerical example is related to Mirab Pipe Company. This company manufacturers polyethylene pipes with 5 sub-affiliates, all of which are centrally managed. The results demonstrated the changes needed in the inputs to maintain the efficiency of the entire system at a constant value based on the changes in the system outputs.

Keywords: Inverse Data Envelopment Analysis, Resource Allocation, Centralized Systems

Introduction

Data envelopment analysis is a non-parametric measure for the determination of the efficiency (performance) of an organization, which was developed by Charans et al in 1978. Data envelopment analysis is used to evaluate the relative efficiency of decision-making units using mathematical programming. The measure of efficiency of a unit is a function of various factors including the number of units, level of unit input, level of unit output, number of components of unit input, the number of components of the output unit, the type of production technology and the model used (Charans et al., 1978). In some researches, researchers seek to determine some effective parameters such as the level of input or output of a decision-making unit such that the measure of efficiency of the unit equals a certain amount. Thus, a group of such issues is called inverse data envelopment analysis in the literature. In data envelopment analysis, the goal is to determine efficiency, but in inverse data envelopment analysis, the best possible output (input) for the assumed input (output) is investigated under conditions where the value of the target function of the original data envelopment analysis model remains constant. As well, when output is to be produced at a certain level, to what degree input should be consumed to maintain or improve efficiency (Zhang and Su, 1999).

It is in this case that the issue of inverse data envelopment analysis is viewed as the most important method to administer a strategy to maintain the efficiency of any organization as it allows organizations to make new decisions in the area of their activities with the same level of efficiency. In today's world, considering the advancement of technology, we are observing states in which all decision-making units are supervised by a central unit that by itself monitors the efficiency of other units (such as Wal-Mart hypermarkets, banks, chain restaurants, McDonald stores, etc.). This situation occurs when all units are affiliated with a central organization, which provides the units with necessary resources and also monitors the entire consumption of different inputs and production of outputs. Thus, this method is called centralized resource allocation (Lozano and Villa, 2004). It is however assumed that all decision-making units are supervised by a central unit. In centralized systems, we often face situations where we have to increase or decrease output because of internal pressures (such as insufficient workforce, employee motivation, managerial policy change, etc.) or external pressures (such as sanctions, exports, etc.) on the system. These factors certainly affect the consumption of inputs, management strategies, etc. In this article, resource allocation was performed in a centralized system using inverse data envelopment analysis.

In the present study, it is investigated to what degree any changes made to the outputs (either compulsory or voluntary) can affect the system and the way it works, and how these changes can be managed. In addition, it is demonstrated to what degree these changes affect the inputs of the entire system and how and to what degree this possible change in the level of inputs is distributed among the decision-making units in a centralized system. This reveals how the maximum possible efficiency is maintained, not only for each unit, but also for the entire system. The proposed model is solved by a numerical example using GAMS software. The numerical example is the Toos Mirab Pipe Company which is a manufacturer of polyethylene pipes, with 5 sub-affiliates, all of which are centrally managed. They are also homogeneous, meaning that they all do the same thing. This centralized system has 4 inputs (manpower, raw materials, working hours and machinery) and three outputs (water supply pipe, lateral irrigation pipe and drip tape).

Theoretical foundations

- **Data Envelopment Analysis**

Suppose that there are n decision-making units, each with m input and s output, and that DMUo, where o∈ {1,2,…., N}, is the unit whose efficiency estimation is under investigation. The vectors $X_j = (x_{1j}, x_{2j},…, x_{mj})^t$ and $Y_j = (y_{1j}, y_{2j},…, y_{sj})^t$ are the input and output vectors corresponding to DMUj, respectively. In data envelopment analysis, it is usually assumed that all inputs and outputs are positive. Also, it is possible to produce one of the basic concepts in calculating efficiency and is defined as follow:

$P_{DEA} - \{(X,Y) \mid$

An output vector Y is produced by the input vector X

- **Basic models in data envelopment analysis**

In data envelopment analysis, mathematical programming is utilized to analyze the efficiency of decision-making units and ascribe a measure to each decision-making unit as a measure of efficiency. In other words, in data envelopment analysis, a decision-making unit is compared with a set of efficiency points that constitute the efficiency boundary so that it is evaluated. Hence, based on a set of production possibilities, various models can be proposed for this purpose. Generally speaking, these models are divided into two radial and non-radial categories. In radial models, the level of changes in inputs or outputs is appropriate, but in non-radial models, the level of changes in each of the inputs or in each of the outputs is independent of each other (Zafari, Ayoob 2016).

- ✓ **Radial models in data envelopment analysis**

Radial models are divided into two important categories based on appropriate contraction at the inputs or appropriate expansion at the outputs.

A) Radial based models at the input

Radial models of nature input seek to find a virtual decision-making unit on the boundary of efficiency whose output is not less than the output of the decision-making unit under evaluation and whose input is proportionally less than the input of the decision unit under evaluation. If such a virtual decision-making unit is found, the evaluated decision-making unit is called inefficient and otherwise efficient.

Based on the feasibility of production, different models can be considered to determine the measure of efficiency in the unit under the DMUo study. The envelopment form of some of these models is in the form of relation 1.

Relation 1

$$\theta_o^* = \min\ \theta$$
$$s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}\ , i = 1, \dots, m$$
$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}\ , r = 1, \dots, s$$
$$\lambda \in \Omega,$$

B) Radial based models at the output nature

Radial models of output nature seek to find a virtual decision-making unit on the efficiency boundary whose input is not less than the input of the decision-making unit under evaluation and whose output is appropriately higher than the output of the evaluated decision-making unit. If such a virtual decision unit is found, the evaluated decision-making unit is called inefficient and otherwise efficient. The envelopment form is the nature of the output in relation 2.

Relation 2

$$\varphi_o^* = \max\ \varphi$$
$$s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}\ , i = 1, \dots, m$$
$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi y_{ro}\ , r = 1, \dots, s$$
$$\lambda \in \Omega,$$

Model φ_o^* و θ_o^* is called measure of efficiency in the DMU_0 at the input and output nature, resperctively.

- ✓ **Non-radial based models in data envelopment analysis**

Another category of models of different measures gauges the distance of the unit under evaluation with the efficiency boundary and are introduced as an efficient or inefficient unit based on the measure resulting from the unit under evaluation. Some researchers have proposed various non-radial models for calculating a measure of efficiency, such as additive models, SBM, and Russell advanced measure (Pasteur et al., 1999).

- **Inverse envelopment data**

The inverse data envelopment analysis having been introduced, he and his colleagues asked the following question:

Question 1 (Output estimation): If certain inputs from DMUo are increased to a certain extent from among a group of decision-making units, how is the outputs of this unit increased for the efficiency of this new unit to be maintained compared to other units?

To answer the question, suppose that the DMUo inputs are increased from X_o to $\alpha_o = X_o + \Delta X_o$ such that $\Delta X_o \geq 0$ and $\Delta X_o \neq 0$. We seel to estimate the output vector (β_o^*) so that the measure of efficiency remains constant at the same level (φ_o^*).

To answer the basic question (output vector estimation), the problem of multi-objective linear programming 2-6 IS proposed by him and his colleagues:

Model 2-6

$$
\begin{aligned}
&\max\ (\beta_{1o}, \dots, \beta_{so}) \\
&s.t.\ \ \sum_{j=1}^{n} \lambda_j x_{ij} \leq \alpha_{io}\ , i = 1, \dots, m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi_o^* \beta_{ro}\ , r = 1, \dots, s \\
&\beta_{ro} \geq y_{ro}, \qquad r = 1, \dots, s \\
&\lambda \in \Omega.
\end{aligned}
$$

In this model, $(\lambda, \beta_o) \in \mathbb{R}^n \times \mathbb{R}^s$ is the variables vector. Also, φ_o^* is the optimal value of the linear programming problem (Wi et al. 2000).

Generalization of the first question: If certain inputs from DMU_o are increased to a certain level among a group of decision-making units, then how are the outputs of this unit increased for the efficiency of this new unit to be maintained compared to with other units?

In order to answer this question, Jahanshahloo and his colleagues introduced the problem of multi-objective linear programming 2-7:

Model 2-7:

$$
\begin{aligned}
&\max\ (\beta_{1o}, \dots, \beta_{so}) \\
&s.t.\ \ \sum_{j=1}^{n} \lambda_j x_{ij} \leq \alpha_{io}\ , i = 1, \dots, m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \geq (1 + \frac{\eta}{100}) \varphi_o^* \beta_{ro}\ , \qquad r = 1, \dots, s \\
&\beta_{ro} \geq y_{ro}, \qquad r = 1, \dots, s \\
&\lambda \in \Omega.
\end{aligned}
$$

In the model above, $(\lambda, \beta_o) \in \mathbb{R}^n \times \mathbb{R}^s$ is the variables vector and φ_o^* is the optimal value opf the linear programming problem with η being the constant value determined by the decision maker, as $0 \leq \eta \leq \frac{\varphi_o^*-1}{\varphi_o^*} * 100$. It is also obvious that if $\eta = \frac{\varphi_o^*-1}{\varphi_o^*} * 100$, then the new unit will be efficient (Jahanshahloo et al., 2004).

Second question (Input problem estimation): If certain outputs of DMU_o from among a group of decision-making units are increased to a certain level, the question is how are the inputs of this unit increased for the efficiency of this new unit to be maintained in comparison with other units?

In order to answer the above question, let's suppose DMU_0 outputs increases from Y_0 to $\beta_o = Y_o + \Delta Y_o$, such that $\Delta Y_o \geq 0$ and $\Delta Y_o \neq 0$. In this case, we seek to estimate the input vectors (α_o^*) $(Hadi\ Vincheh\ et\ al.\ 2008)$. Thus, in reality,

$$\alpha_o^* = (\alpha_{1o}^*, \alpha_{2o}^*, \dots, \alpha_{mo}^*)^t = X_o + \Delta X_o, \qquad \Delta X_o \geq 0$$

In order to answer the second question (input vector estimation), the problem of multi-objective linear programming 2-8 was proposed by Hadi Vincheh et al.

Model 2-8

$$\begin{aligned}
&\min\ (\alpha_{1o}, \alpha_{2o} \dots, \alpha_{mo}) \\
&s.t.\ \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o^* \alpha_{io}, \qquad i = 1, \dots, m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \geq \beta_{ro}, \qquad r = 1, \dots, s \\
&\alpha_{io} \geq x_{io}, \\
&\lambda \in \Omega.
\end{aligned}$$

In the above model, $(\lambda, \alpha_o) \in \mathbb{R}^n \times \mathbb{R}^m$ is the variabke vector. Also, θ_o^* os the optimal value of the linear programming problem (Hadi Vincheh et al. 2008).

It should be noted that Jahanshahloo and his colleagues raised the above question in more detail:

Generalization of the second question: If certain outputs of DMUo are increased to a certain extent from among a group of decision-making units, then how are the inputs of this unit increased for the efficiency of this new unit to be maintained in comparison with other units?

In order to answer this question, Jahanshahloo and his colleagues raised the problem of multi-objective linear programming model 2-9:

Model 2-9:

$$\begin{aligned}
&\min\ (\alpha_{1o}, \dots, \alpha_{mo}) \\
&s.t.\ \sum_{j=1}^{n} \lambda_j x_{ij} \leq (1 + \frac{\eta}{100}) \theta_o^* \alpha_{io}, \qquad i = 1, \dots, m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \geq \beta_{ro}, \qquad r = 1, \dots, s \\
&\alpha_{io} \geq x_{io}, \quad i = 1, \dots, m \\
&\lambda \in \Omega.
\end{aligned}$$

In the model above, $(\lambda, \alpha_o) \in \mathbb{R}^n \times \mathbb{R}^m$ is the vector of the variables and θ_o^* is the optimlaa value of the linear programming problem with η being the constant value specified by thee decion making unit. Thus, $0 \leq \eta \leq \frac{1-\theta_o^*}{\theta_o^*} * 100$. It is also clear that if $\eta = \frac{1-\theta_o^*}{\theta_o^*} * 100$, then, the new unit will be made efficient (Jahanshahloo et al., 2004).

Research background

Table 1: Review of some of the past researches on resource allocation using data envelopment analysis

Research title	Authors	Main objectives
Human resource rights using centralized data envelopment analysis: evidence from Taiwan airport	Ming Min Yu et al. 2013	Human resource reallocation, centralized data envelopment analysis, airport, regular employee, contract employee
Resource allocation using data envelopment analysis according to environmental factors	Ji Woo et al. 2013	Data envelopment analysis, allocation of a specific source, maximizing the desired total output, minimizing the total undesirable output
Centralized resource allocation using data envelopment analysis based on efficiency with limited information	Lee Fang 2017	Inverse data envelopment analysis, for resource allocation in centralized system, output improvement
Centralized resource allocation and goal setting using data envelopment analysis model	Pingchan et al. 2018	Data envelopment analysis, input resource Allocation, output objectives decision making, centralized system
Resource planning in the Chinese Commercial Banking System using mutual envelopment Analysis pertaining to two-step information with unfavorable outputs	Kingjian et al. 2019	Envelopment analysis of two-stage inverse data, undesirable outputs, resource program design

Proposed model for centralized resource allocation

In the proposed inverse data envelopment analysis, if the measure of efficiency of the decision-making unit under evaluation (DMU0) remains unchanged at the same level (the measure of efficiency of the DMUo unit in the nature of input) while its output increases, we want to understand to what degree the inputs of the unit under evaluation should increase. Put it differently, to what degree additional resources should be allocated to decision-making units to create such a system with new outputs, provided that the level of system efficiency is maintained.

Let's suppose that the DMU_0 outputs rise from y_0 to $\beta_0 = y_0 + \Delta y_0$, thus we seek to estimate the input vector (α_0) assuming that the measure of the efficiency remains unchanged at the same level (Mohammadi and Berati, 2016).

The proposed inverse envelope analysis model is in fact a SBM additive envelopment analysis model. In addition to being non-radial, this model has the advantage that it is neither input-oriented nor output-oriented, rather a model that simultaneously takes into account the reduced inputs and outputs (Jahanshahloo et al. 2011).

To evaluate the efficiency of the inverse data envelopment analysis model, model 1 was used in which the variables are as follows:

Xij: The input value of the i used

Yrj: The output value of r produced by DMUj

θ: Factor of radial reduction achievable for inputs

s_i: Input deficit variable i

I = 1, .., m: Index set of inputs

r = 1, .., s: Index set of output

J = 1, .., n: set of decision units

λ_{jk}: The vector is related to DMUj imaging.

Model 1

$$\min \theta = 1 - \frac{1}{m}\sum_{i=1}^{m} \frac{S_i}{\sum_{j=1}^{n} x_{ij}}$$

$$s.t$$

$$\sum_{j=1}^{n}\sum_{k=1}^{n} \lambda_{jk} x_{ij} + S_i = \sum_{j=1}^{n} x_{ij} \quad \forall i = 1,\dots,m$$

$$\sum_{j=1}^{n}\sum_{k=1}^{n} \lambda_{jk} y_{rj} \geq \sum_{j=1}^{n} y_{rj} \quad \forall r = 1,\dots,S$$

$$\sum_{j=1}^{n} \lambda_{jk} = 1 \quad k = 1,\dots,n$$

$$\lambda_{jk} \geq 0 \quad S_i \geq 0$$

The proposed model is centralized resource allocation as Model 2.

Model 2

$$\min \alpha = \sum_{i=1}^{m} \alpha_i$$

$$s.t$$

$$\sum_{j=1}^{n}\sum_{k=1}^{n} \lambda_{jk} x_{ij} + S_i = \theta_o \left(\sum_{j=1}^{n} x_{ij} + \alpha_i\right) \quad \forall i = 1,\dots,m$$

$$\sum_{j=1}^{n}\sum_{k=1}^{n} \lambda_{jk} y_{rj} - S_r = \sum_{j=1}^{n} y_{rj} + \beta_r \quad \forall r = 1,\dots,S$$

$$\sum_{j=1}^{n} \lambda_{jk} = 1 \quad k = 1,\dots,n$$

θ_0 : The optimal value of the linear programming problem

α_i: Input vector of i

β_r: output surplus variable r

For the proposed general model, limitations are applied based on the company under study. These limitations are presented in Model 3.

Model 3 Limitations of the model

$$0/90\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}x_{4j} \leq \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}y_{1j} + \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}y_{2j} + \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}y_{3j} \leq 0/95\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}x_{4j}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}y_{1j} \geq \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}y_{2j}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}y_{2j} \geq \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}y_{3j}$$

$$\lambda_{jk} \geq 0 \quad S_i \geq 0 \quad S_r \geq 0$$

$$i = 1,\ldots,m \quad r = 1,\ldots,S \quad k = 1,\ldots,n$$

According to Model 3: The first limitation expresses the total output tonnage of the first product (water supply pipe), the second product (lateral irrigation pipe) and the third product (drip tape) is between 90 to 95% of the raw materials tonnage arriving at the Mirab Pipe Company. This 5 to 10 percent discrepancy between the output product and the input raw materials is because of the waste during the production process. Some of the molten material and also several meters of the pipes produced are wasted when changing the mold and beginning the production line. In the production process, a problem may arise for one of the machineries and some of the pipes in the meantime be inappropriate and out of standard as they are added to the production waste. Another main reason for the waste created is power outage causing the production to stop. The second and third limitations indicate that the output tonnage of the first product must be greater than the second product and the second product. In other words, for example, if the first product (water supply pipe) produces 100 tons per month, the second product (lateral irrigation pipe) and the third product (drip tape) will definitely be less than this 100 tons.

Research findings

In this section, results of the GAMS software are provided under states of changing orders by the said company which includes 5 decision making units. These units are supervised by a central unit that handles all branches and resource allocation among them. The model has 4 inputs and 3 outputs. The inputs include: manpower, working hours, number of machinery and polyethylene material and the outputs are water supply pipes, lateral irrigation pipes and drip tape.

Table 2: Data related to one month orders of Toos Mirab Pipe Company

Decision making units (DMUj)	Manpower(X1)	Working hours (X2)	Number of machinery(X3)	polyethylene material (X4)	Water supply pipe (Y1)	Lateral irrigation pipes (Y2)	Drip tape (Y3)
DMU01	23	12	5	100	73	19	5
DMU02	7	12	2	20	14	4	1
DMU03	25	12	6	110	80	21	5
DMU04	30	12	7	150	110	29	7
DMU05	18	12	4	90	66	1	4
Total decision units	103	60	24	470	343	74	22

We see that the company is currently producing 343 tons of water supply pipes, 74 tons of lateral irrigation pipes and 22 tons of drip tape in 5 decision-making units with a total of 103 employees, 24 production lines and 470 tons of raw materials per month via 12-hour work shifts. The current efficiency of the company is 0.98. To improve efficiency, it is better to change the resource allocation as summarized in Table 3.

Table 3: Resource reallocation to achieve 100% efficiency

Decision making units (DMUj)	Manpower(X1)	Working hours (X2)	Number of machinery(X3)	polyethylene material (X4)	Water supply pipe (Y1)	Lateral irrigation pipes (Y2)	Drip tape (Y3)
DMU01	17	12	4	77	55.8	10.8	3.55
DMU02	28	12	6	138	101.44	23.56	6.42
DMU03	28	12	6	138	101.44	23.56	6.42
DMU04	21	12	5	96	70.31	12.09	4.62
DMU05	7	12	2	20	14	4	1
Primary system	103	60	24	470	343	74	22
New system	101	60	23	469	343	74	22

One can see that in the reallocation process, the company's efficiency improved to 1. The workforce, the production lines, and the consumption of raw materials saw declines by 2 people, 1 line and 1 ton respectively, which is in the interest of the company.

2- In this case, we considered the output of the first product to increase by 50 tons, the second product by 5 tons and the third product by 2 tons. The output of GAMS software for this case is shown in Table 4

Table 4: Reallocation to increase 50 tons of first output, 5 tons of second output and 2 tons of third output

Decision making units (DMUj)	Manpower(X1)	Working hours (X2)	Number of machinery(X3)	polyethylene material (X4)	Water supply pipe (Y1)	Lateral irrigation pipes (Y2)	Drip tape (Y3)
DMU01	24	12	5	119	86.95	14.33	4.62
DMU02	8	12	2	24	17.14	3.82	0.37
DMU03	27	12	6	137	100.51	22.96	5.54
DMU04	24	12	5	119	86.95	14.33	4.62
DMU05	28	12	6	138	101.44	23.56	5.61
Primary system	103	60	24	470	343	74	22
New system	111	60	24	537	393	79	24

To increase this level of output of the company's products, it is needed to:

1. Increase the workforce by 8 (from 103 to 111). A distribution of the changes in the number of labor force, in comparison with the initial table 2, should be as follows: 1 person should be added to the first unit, 1 person to the second unit, 2 people to the third unit and 10 people to the fifth unit with 6 people be reduced from the fourth unit.
2. Add 67 tons of consumption raw materials to the system. A distribution of the changes in raw materials, compared to Table 2 should be as follows: 19 tons to the first unit, 4 tons to the second unit, 27 tons to the third unit and 48 tons to the fifth unit with 31 tons be reduced from the fourth unit.
3. Compared to Table 2, one can see that the working hours and the number of machines in the original and new systems did not differ.

3- In this case, we considered the output of the first product to increase by 50 tons, the second product by 30 tons and the third product by 5 tons. The output of GAMS software for this case is shown in Table 5.

Table 5: Reallocation to increase 50 tons of first output, decrease of 30 tons of second output and decrease of 5 tons of third output

Drip tape (Y3)	Lateral irrigation pipes (Y2)	Water supply pipe (Y1)	polyethylene material (X4)	Number of machinery(X3)	Working hours (X2)	Manpower (X1)	Decision making units (DMUj)
DMU01	24	12	5	119	86.95	14.33	5.43
DMU02	24	12	5	119	86.95	14.33	5.43
DMU03	24	12	5	119	86.95	14.33	5.43
DMU04	24	12	5	119	86.95	14.33	5.43
DMU05	14	12	3	62	45.19	7.84	2.88
Primary system	103	60	24	470	343	74	22
New system	110	60	23	538	393	44	17

To increase the output of the first product by 50 tons, decrease the second product by 30 tons and decrease the third product by 5 tons, it is needed to:

1. Increase the number of labor force by 7 (from 103 to 110). A distribution of the changes in the number of labor force, compared to the initial table 2 should be as follows: 1 person should be added to the first unit, 17 people to the second unit and also 1 person to the third unit, 6 people to the fourth unit with 4 people reduced from the fifth unit.

2. Add sixty-eight tons of raw materials to the system. A distribution of this level of changes in raw materials, compared to Table 2, should be as follows: 19 tons to the first unit, 99 tons to the second unit, 9 tons to the third unit and 28 tons to the fifth unit with 31 tons reduced from the fourth unit.

3. Reduce one unit of machinery (Three machines should be added to the second unit and 1 machine be reduced from the third unit, and also 2 machines be reduced from the fourth unit and 1 machine from the fifth unit).

4. Compared to Table 2, one can see that the working hours in the original and new systems did not differ.

4- In this case, we considered the output of the first product to reduce by 60 tons, the second product by 11 tons and the third product by 5 tons. The output of GAMS software for this case is shown in Table 6.

Table 6: Reallocation to reduce 60 tons of first output, 11 tons of second output and 5 tons of third output

Decision making units (DMUj)	Manpower(X1)	Working hours (X2)	Number of machinery(X3)	polyethylene material (X4)	Water supply pipe (Y1)	Lateral irrigation pipes (Y2)	Drip tape (Y3)
DMU01	28	12	6	138	101.44	23.56	6.42
DMU02	28	12	6	138	101.44	23.56	6.42
DMU03	26	12	6	127	92.98	21/66	5.89
DMU04	7	12	2	20	14	4	1
DMU05	7	12	2	20	14	4	1
Primary system	103	60	24	470	343	74	22
New system	96	60	22	443	283	63	17

To reduce the output of the first product by 60 tons, reduce 11 tons of the second product and reduce the third product by 5 tons, it is required to:

1. Reduce the number of labor force (from 103 to 96). A distribution of the changes in the number of labor, compared to the initial table 2 should be as follows: 5 people should be reduced from to the first unit, 21 be added to the second unit and 1 person the third unit while 23 and 11 people be reduced from the fourth and fifth unit, respectively.

2. Reduce 27 tons of consumption raw materials from the system. A distribution of this level of changes in raw materials, compared to Table 2 should be as follows: 38 tons should be added to the first unit, 118 tons to the second unit, 17 tons to the third unit while 130 tons be reduced from the fourth unit and 70 tons from the fifth unit.

3. Reduce two units of machinery (1 machine is to be added to the first unit and 4 machines to the second unit while 5 machines are to be reduced from the fourth unit, with 2 machines from the fifth unit).

4. Compared to Table 2, one can see that the working hours in the original and new systems did not differ.

Conclusions and Suggestions

Data envelopment analysis is a method to evaluate the relative efficiency of a decision-making unit compared to other units that makes use of the same inputs to produce the same outputs. One of the most widely used issues in data envelopment analysis is the changing of one index and determination of other indicators to maintain or improve efficiency. This is reflected under the inverse data envelopment analysis. In this paper, we discuss various types of inverse data envelopment analysis models while maintaining efficiency and improving it through centralized resource allocation models.

In this paper, resource allocation in a centralized system was performed using inverse data envelopment analysis. The centralized resource allocation model was provided to the Mirab Pipe Company. This company manufacturers polyethylene pipes with 5 sub-affiliates (units 1 and 2 in Mashhad, units 4 and 3 in Isfahan and unit 5 in Ahvaz), all of which are managed centrally as they are also homogeneous in the sense that they all do the same job. This centralized system has 4 inputs (manpower, raw materials, working hours and machinery) and 3 outputs (water supply pipe, lateral irrigation pipe and drip tape). Various states of system outputs (increased or decreased volume) were considered for the proposed model as it was based on GAMS software. Based on that, system inputs were specified. Based on changes in system outputs, The results demonstrated the changes needed in the inputs to maintain the efficiency of the entire system at a constant value. The findings of this thesis allow Mirab Pipe's directors to be aware of the results under complicated situations, before they get down to administer their plan to increase or decrease orders, so to make the right decision with an open mind.

The work done in the present study can also be performed for decentralized systems in the future researches. As well, various models of data envelopment analysis not yet combined with other issues in statistics or research in action can be applied to achieve an optimal result. Hence, more useful and at the same time more practical procedures can be utilized for different evaluations. For example, data envelopment analysis can be used via linear and non-linear programming as well as correctional processes in statistics. This is while, the proposed model can be combined by considering the probabilistic and fuzzy numbers so that more recent models can be used to make more accurate assessments concerning decision-making units.

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